$\lim_{x \to \pi} \frac{(\cos x)}{(x-\pi)^2} = \frac{-1}{0} = \infty$ DNE

$\frac{-1}{0}$ $\frac{1}{0}$ 42

9:30 → 11:00

Second

جامعة بيرزيت

BIRZEIT UNIVERSITY

2nd semester 2012/2013

Name: [redacted]

MATH 331

Number: [redacted]

2nd Exam

[redacted]

Problem#1(24 points) Solve the following problems then circle the most correct answer

1- Consider the differential equation $(x-\pi)^2 y'' + (\cos x)\, y' + (\sin x)\; y = 0$. The ponit $x = \pi$ is

a) an ordinary point b) singular ordinary point c) singular regular point

d) singular irregular point e) none of the above

$x = 0$ singular

2- Consider the differential equation $x^2 y'' + xy' + (x+1)y = 0$.

A series solution of the following form can be used

a) $\sum_{n=0}^{\infty} a_n (x+1)^n$ b) $\sum_{n=0}^{\infty} a_n x^n$ c) $\sum_{n=0}^{\infty} a_n x^{n+r}$

d) a and c e)none of the above.

$\sum_{}^{\infty} a_n (x-1)^n$

$\lim_{x \to 0} x^2 \cdot \frac{x}{x^2}$

$\lim x \cdot \frac{x+1}{x^2}$

regular

3- For the following IVP

$x^2 y'' + x y' - y = 0$, $y(1) = 1$ $y'(1) = \beta$.

The value of β that makes the solution $y(x) \to 0$ as $x \to 0$ is

a) -1 b) 1 c) 2 d) -2 e) none of the above

$r^2 - 1 = 0$

$r^2 = 1$

$r = \pm 1$

$y(x) = c_1 x^{-1} + c_2 x$

$\boxed{\text{we need } c_1 = 0}$

$-\frac{(s^2+9-2s^2)}{(s^2+9)^2} = \frac{2s^2-s^2-9}{(s^2+9)^2}$

$-\left(\frac{s}{s^2+9}\right)' = -\left(\frac{(s^2+9)-2s^2}{(s^2+9)^2}\right)$

4- The Laplace transform of $f(t) = t\cos(3t)$ is

a) $\frac{9-s^2}{(9+s^2)^2}$ b) $\frac{s^2-9}{(9+s^2)^2}$ c) $\frac{s^2-9}{9+s^2}$ d) $\frac{9-s^2}{9+s^2}$ e) none of the above

$y(1) = c_1 + c_2 = 1$

$y'(x) = -c_1 x^{-2} + c_2$

$\mathcal{L}\{t \cos(3t)\}$

$-\left(\frac{s}{s^2+9}\right)' = -\left[\frac{(s^2+9)-2s^2}{(s^2+9)^2}\right]$

$y'(1) = -c_1 + c_2 = \beta$

$-\left(\frac{s}{s^2+9}\right)'$ $\frac{s^2-9}{(s^2+9)^2}$

$e^t * t^2$

$= \frac{(s^2+9) - s(2s)}{(s^2+9)^2}$

$e^t - 2t e^t + 2e^t$

5- If $f(t) = e^t$ and $g(t) = t^2$ Then $(f * g)(1) =$

a) $3 + 2e$ b) $3 - 2e$ c) $2e - 5$ d) 0 e)none of the above

$\int_0^t e^t \cdot t^2 \, dt$

$t^2 + e^t$

$2t \to e^t$

$2 \to e^t$

$0 \to e^t$

$-\left(\frac{9-s^2}{(s^2+9)^2}\right)$

$2c_2 = 1 + B$

$c_1 = \frac{1-B}{2} = 0$ $\boxed{B = 1}$ $= \frac{s^2-9}{}$

$= c_1 + 1 + B$

$A(s-1)+Bs=1$

$B=1$

$s^2\mathcal{L}\{y\}-sy(0)-y'(0)-\mathcal{L}\{y\}=\mathcal{L}\{u_3(t)\}$

$s^2\mathcal{L}\{y\}-\mathcal{L}\{y\}=\frac{e^{-3s}}{s}$

$\mathcal{L}\{y\}=\frac{e^{-3s}}{s(s^2-1)}$

$y(t)=\mathcal{L}^{-1}\left(\frac{e^{-3s}}{s(s^2-1)}\right)$

$\Rightarrow \mathcal{L}^{-1}\left(\frac{e^{-3s}}{s}\right)*\mathcal{L}^{-1}\left(\frac{1}{s^2-1}\right)$

6- If $y(t)$ is the solution of the IVP

$y''-y=u_3(t)$, $y(0)=0$ $y'(0)=0$

Then $y(2)=$

a) $cosh\,(1)$ b) $sinh(1)+cosh(1)$ c) $sinh\,(1)$ d) 0 e) none of the above

$1 * \sinh t$

$\frac{1}{s(s-1)}=\frac{A}{s}+\frac{B}{s-1}$, $(-1+e^{t})\Rightarrow u_3(t)$

$s^2\mathcal{L}\{y\}+\frac{1}{4}\mathcal{L}\{y\}=\mathcal{L}\{\delta(t-1)\}$

7- If $y(t)$ is the solution of the IVP

$y''+\frac{1}{4}y=\delta(t-1)$ $y(0)=0$ $y'(0)=0$

$y(t)=\mathcal{L}^{-1}\left(\frac{e^{-s}}{s^2+\frac{1}{4}}\right)$

Then $y(\frac{\pi}{2}+1)=$

a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) $\frac{1}{2\sqrt{2}}$ d) 2 e) none of the above

$\mathcal{L}(y)\left(s+\frac{1}{4}\right)=\frac{e^{-s}}{s+\frac{1}{4}}=\frac{e^{-s}}{s^2+(\frac{1}{2})^2}$

$\frac{1}{2}\mathcal{L}^{-1}\left(\frac{\frac{1}{2}e^{-s}}{s^2+\frac{1}{4}}\right)$

$\left(\frac{1}{2}\sin\frac{1}{2}t\right)u_1(t)$

$\frac{1}{2}\sin\frac{1}{2}\left(\frac{\pi}{2}+1\right)$

$u_1(t)\left[e^{-t(t-1)}\right]$

$\frac{1}{2}\left[\sin\left(\pi+\frac{1}{2}\right)\right]$?

$\sin(A+B)=\sin\pi\cos\frac{1}{2}+\cos\pi\sin\frac{1}{2}$

$=0+-1\times$

8- Let x_0 be an ordinary point of the differential equation $y''+p(x)y'+q(x)y=0$. Suppose the radius of convergence of $p(x)$ about x_0 is 2 and the radius of convergence of $q(x)$ about x_0 is 3 then the series solution radius of convergence about x_0 is

a) ∞ b) ≥ 3 c) ≤ 2 d) ≥ 2 e) none of the above

$\frac{1}{2\sqrt{2}}$

$\rho \geq 2$

$\frac{1}{2}\sin\left(\frac{\pi}{4}+\frac{1}{2}\right)$

$\frac{1}{2}\sin\left(\frac{2\pi+1}{8}\right)$

$\frac{1}{2}\sin\left(\frac{\pi}{4}+\frac{1}{4}\right)$

$\frac{1}{2}\sin\frac{1}{2}(t)$

$\frac{1}{2}\sin\left(\frac{1}{2}(t-1)+\frac{1}{2}\right)$

$\frac{1}{2}\sin\frac{1}{2}(t-1)$

9- Consider the series solution $y(x) = \sum_{n=0}^{\infty} a_n (x-1)^n$ of the IVP

$$y'' - e^{x-1}y' + 3y\cos(x-1) = 0, \qquad y(1) = 1 \qquad y'(1) = 2.$$

-2 $3 \times 1 \times 1$

then $a_2 =$

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 5 d) $\frac{5}{2}$ e) none of the above

$a_2 = \frac{y''(1)}{2!}$

10- The Laplace transform of $f(t) = u_2(t) * t^2$ is

a) $\frac{e^{-2s}}{s^3}$ b) $\frac{e^{-2s}}{s^4}$ c) $2\frac{e^{-2s}}{s^3}$ d) $2\frac{e^{-2s}}{s^4}$ d) none of the above

$\mathcal{L}\{10\} = (\delta)\, 10$

$\frac{e^{-2s}}{s} \cdot \frac{1}{s^3}$

$-1\left(e^{-2s}\right)'''$

11- The Laplace transform of $f(t) = \delta(t-2)\, t^3$ is

a) $8e^{-s}$ b) $8e^{-2s}$ b) s^3e^{-2s} b) $8e^{-2t}$ e) none of the above

$\mathcal{L}\{\delta(t-2)\, t^3\} = e^{-2s} f(c)$

$= e^{-2s} (-2)^3$

$= 8 e^{-2s}$

12- Suppose that Laplace transform of $f(t)$ is $\frac{s e^{-\pi s}}{s^2+4}$ then $f(2\pi)$ is

a) -1 b) 1 c) 0 d) 2 e) none of the above

$\cos 2(t-\pi)$

$\cos 2(t-\pi)$

$\cos 2(2\pi - \pi)$

$\cos 2(\pi) = 1$

Problem #2)(8 points) The polynomial solution of

$$(1-x^2)y'' - 2xy' + 6y = 0$$ is

Serm Solut araund $x_0 = 0$

$y(x) = \sum_0^\infty a_n x^n$, $y'(x) = \sum_1^\infty a_n n x^{n-1}$, $y''(x) = \sum_2^\infty a_n n(n-1) x^{n-2}$

substitud in the equ

$$(1-x^2)\sum_2^\infty n(n-1)a_n x^{n-2} - 2x\sum_1^\infty n a_n x^{n-1} + 6\sum_0^\infty a_n x^n = 0$$

$$\sum_2^\infty n(n-1)a_n x^{n-2} - \sum_2^\infty n(n-1)a_n x^n - \sum_1^\infty 2n a_n x^n + \sum_0^\infty 6a_n x^n = 0$$

$$\sum_0^\infty (n+2)(n+1)a_{n+2} x^n - \sum_2^\infty n(n-1)a_n x^n - \sum_1^\infty 2n a_n x^n + \sum_0^\infty 6a_n x^n = 0$$

$$(2a_2 + 6a_0)x^0 + (6a_3 - 2a_1 + 6a_1)x^1 + \sum_2^\infty \left[(n+2)(n+1)a_{n+2} - (n(n-1) + 2n - 6)a_n\right]x^n = 0$$

$2a_2 + 6a_0 = 0 \Rightarrow 2a_2 = -6a_0 \Rightarrow \boxed{a_2 = -3a_0}$ // $6a_3 + 4a_1 = 0$, $6a_3 = -4a_1 \Rightarrow \boxed{a_3 = -\frac{2}{3}a_1}$

$(n+2)(n+1)a_{n+2} - (n(n-1) + 2n - 6)a_n = 0 \Rightarrow (n+2)(n+1)a_{n+2} = (n^2+n-6)a_n$

$\Rightarrow (n+2)(n+1)a_{n+2} = (n-2)(n+3)a_n \Rightarrow \boxed{a_{n+2} = \frac{(n-2)(n+3)a_n}{(n+2)(n+1)}}$ recurrence relation

$n=2 \Rightarrow a_4 = 0$

~~$n=3 \Rightarrow a_5 = \frac{1 \cdot 6 a_3}{5 \cdot 4}$~~ $= \frac{6}{20}a_3$ $= \frac{6 \cdot 2}{10 \cdot 3}a_1 = -\frac{4}{10}a_1$ // $n=4 \Rightarrow a_6 = 0$ all even $n > 2$ $a_n = 0$

$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots$

$$\boxed{y(x) = a_0\left[1 - 3x^2\right] + a_1\left[x + 4x^3 - \frac{4}{10}a_1 x^5 + \cdots\right]}$$

Answer

$\frac{1}{2}\sin\left(\frac{2\pi}{4} - \frac{\pi}{4}\right)$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -5 & -2 \end{array}\right]$$

$y(x) = 3c_1(x+2)^2 - 2c_2(x+2)^{-3}$

$y(-1) = 3c_1 - 2c_2$

Problem #3(12 points)

a) If $y(x)$ is the solution of the IVP

$(x+2)^2 y'' - 6y = 0,\qquad y(-1) = 1\qquad y'(-1) = 1$

Then $y(0) =$

Let $t = x+2 \Rightarrow dt = dx$

$\Rightarrow t^2 y'' - 6y = 0$ Euler equ

$r^2 + (0-1)r - 6 = 0 \Rightarrow r^2 - r - 6 = 0 \Rightarrow (r-3)(r+2) = 0$

$\boxed{r_1 = 3,\ r_2 = -2}$

$\Rightarrow y(t) = c_1 t^3 + c_2 t^{-2} \Rightarrow \boxed{y(x) = c_1(x+2)^3 + c_2(x+2)^{-2}}$

$y(-1) = c_1 + c_2 = 1$

$y'(-1) = 3c_1 - 2c_2 = 1$

$\boxed{c_2 = \frac{2}{5},\quad c_1 = 1 - \frac{2}{5} = \frac{3}{5}}$

$\Rightarrow y(0) = c_1 2^3 + c_2 2^{-2}$

$= \frac{3}{5} 8 + \frac{2}{5}\cdot\frac{1}{4}$

$\frac{24}{5} + \frac{1}{10} = \frac{49}{10}$

Answer: $\frac{49}{10}$

b) The Laplace transform of $f(t) = u_1(t)\sin(2t)$ is

$f(t) = u_1(t)\sin(2(t-1+1)) = u_1(t)\sin(2(t-1)+2) = u_1(t)[\sin(2(t-1))\sin 2 + \cos(2(t-1))$

$\Rightarrow \mathcal{L}\{f(t)\} = \mathcal{L}\{u_1(t)\sin(2(t-1))\sin 2 + u_1(t)\cos(2(t-1))\cos(2)\}$

$= \sin 2\left[\frac{2e^{-s}}{s^2+4}\right] + \cos(2)\frac{s e^{-s}}{s^2+4}$

Answer: $\sin 2\left[\frac{2e^{-s}}{s^2+4}\right] + \cos(2)\left[\frac{se^{-s}}{s^2+4}\right]$

c) Suppose that Laplace transform of $f(t)$ is $\frac{(1+3s)\,e^{-s}}{(s-2)^2+9}$ then $f(2)$ is

$f(t) = \mathcal{L}^{-1}\left\{\frac{(1+3s)e^{-s}}{(s-2)^2+9}\right\} = \mathcal{L}^{-1}\left\{\frac{e^{-s}}{(s-2)^2+9}\right\} + 3\mathcal{L}^{-1}\left\{\frac{se^{-s}}{(s-2)^2+9}\right\}$

$= \frac{u_1(t-1)}{3}\sin 3(t-1)\cdot e^{2(t-1)} + 3u_1(t-1)\cos 3(t-1)\, e^{2(t-1)}$

$\cancel{f(2) = \frac{1}{3}\sin(3)e^2 + 3\cos(3)e^2}$

Answer:

$u_1(t-1)e^{2(t-1)}\left\{\frac{1}{3}\sin(3(t-1)) + 3\cos(3(t-1))\right\}$

$F(S)$

Problem #4)(6 points) Show that $L\{u_a(t)\,f(t-a)\} = e^{-as}L\{f(t)\}$ where $L\{f(t)\}$ is the Laplace transform of $f(t)$.

$$\mathcal{L}[u_a(t) f(t-a)] = \int_0^\infty e^{-st} \cdot u_a(t)\, f(t-a)\, dt$$

$(t-a) = g$

$dt = dg$

$t=a,\ g=0$

$t=\infty \Rightarrow g=\infty$

$$= \int_a^\infty e^{-st} f(t-a)\, dt$$

$$= \int_0^\infty e^{-s(g+a)} \cdot f(g)\, dg$$

$$= \int_0^\infty e^{-sg} \cdot e^{-sa} f(g)\, dg$$

$$= e^{-as} \int_0^\infty e^{-sg} f(g)\, dg$$

$$= e^{-as} \mathcal{L}\{f(t)\} \quad \#$$

6

solution of second exam (1)

#1

1- $\rho_1 = 2$ $\rho_2 = 3$

Radius of conv ≥ 2

(d)

2. $y(x) = \sum_{n=0}^{\infty} a_n (x-1)^n$

$y'' - e^{x-1} y' + 3y\cos(x-1) = 0$ $y(1) = 1$ $y'(1) = 2$

a_2 ?

$a_0 = 1$ $a_1 = 2$

$a_2 = \frac{y''(1)}{2!} = \frac{-(-2+3)}{2} = \frac{-1}{2}$

(b)

3. $(x-\pi)^2 y'' + (\cos x) y' + (\sin x) y = 0$

$x = \pi$ is singular irregular point

$\lim_{x \to \pi} \frac{\cos x}{(x-\pi)^2} (x-\pi) \to -\infty$

(d)

$\lim_{x \to \pi} \frac{\sin x}{(x-\pi)^2} (x-\pi)^2 \to 0$

4- $x^2 y'' + x y' + (x+1) y = 0$

a and c

at $x=0$ is singular rigular point

(d)

at $x=-1$ is ordinary point

5. $x^2 y'' + xy' - y = 0$ $\quad y(1)=1$ (2) $y'(1)=\beta$

?? $y \to 0$ as $x \to 0$

(b)

Eular equation

$r^2 - 1 = 0$

$r = \pm 1$

Solution $y = c_1 x^{r_1} + c_2 x^{+r_2}$

$y = c_1 x + c_2 x^{-1}$ $\qquad y' = c_1 + c_2 \frac{-1}{x^2}$

$y(1)=1 \Rightarrow c_1 + c_2 = 1$

$y'(1)=\beta \Rightarrow c_1 - c_2 = \beta$

$\Rightarrow c_1 = \frac{1+\beta}{2} \qquad c_2 = \frac{1-\beta}{2}$

$y = \frac{1+\beta}{2} x + \left(\frac{1-\beta}{2}\right)\frac{1}{x}$

$y \to 0$ as $x \to 0$

when $\boxed{\beta = 1}$

5. $f(t) = t \cos 3t$

$\mathcal{L}(f(t)) = -\frac{d}{ds}\left(\frac{s}{s^2+9}\right)$

$= -\left(\frac{s^2+9 - s(2s)}{(s^2+9)^2}\right) = -\left(\frac{9-s^2}{(s^2+9)^2}\right)$

$= \frac{s^2-9}{(s^2+9)^2}$

(b)

7. $f(t)=t$ $g(t)=t^2$ (3)

$(f*g)(1)$

$L(f*g)=L(f)\cdot L(g)=\frac{2}{s^3(s-1)} = \frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{D}{s-1}$

$A=B=C=-2$ $D=2$

$(f*g)=-2-2t-t^2+2e^t$

$(f*g)(1)=2e-5$ (c)

8. $f(t)=u_2(t)\cdot t^2$

$L(f(t))=L(u_2(t))\cdot L(t^2)$

$=\frac{e^{-2s}}{s}\cdot\frac{2!}{s^3}$

$=\frac{2e^{-2s}}{s^4}$ (d)

9. $f(t)=\delta(t-2)t^3$

$L(f(t))=-(e^{-2s})'''$ (b)

$=8e^{-2s}$

10. $L(f(t))=\frac{se^{-\pi s}}{s^2+4}$ $f(2\pi)$

$f(t)=u_\pi(t)\cos 2(t-\pi)$ (b)

$f(2\pi)=1\cos 2(2\pi-\pi)$

$=1$

11. $y''-y=u_3(t)$

?? $y(2)=?$

$s^2L(y) - \cancel{sy(0)} - \cancel{y'(0)} - L(y) = \frac{e^{-3s}}{s}$

$L(y) = \frac{e^{-3s}}{s(s^2-1)} \to u_3(t)$ but $\underline{2<3}$

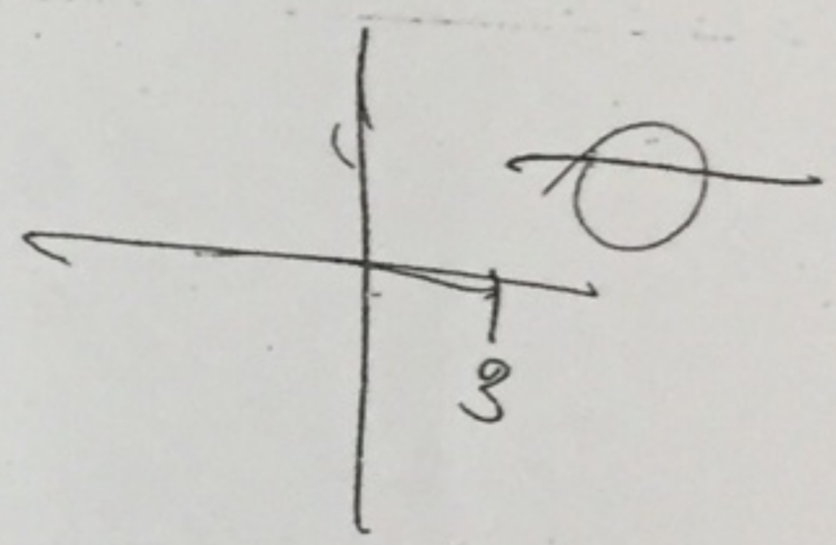
$\Rightarrow u_3(t)$ at $t=2$ equal 0

$y(2)=0$ (d)

12. $y''+\frac{1}{4}y=\delta(t-1)$ $y(0)=0$ $y'(0)=0$

$y(\frac{\pi}{2}+1)$

$s^2L(y) - \cancel{sy(0)} - \cancel{y'(0)} + \frac{1}{4}L(y) = e^{-s}$



$L(y)[s^2+\frac{1}{4}] = e^{-s}$

$L(y) = \frac{e^{-s}}{s^2+\frac{1}{4}}$

$y = 2u_1(t)\sin\frac{1}{2}(t-1)$

$y(\frac{\pi}{2}+1) = 2(1)\sin\frac{1}{2}(\frac{\pi}{2}+1-1)$

$= 2\sin\frac{\pi}{4}$ (b)

$= 2\frac{1}{\sqrt{2}} = \sqrt{2}$

Q2 The Polynomial Solution ? (5)

$(1-x^2)y'' - 2xy' + 6y = 0$

$x=0$ is ordinary point

let $y = \sum_{n=0}^{\infty} a_n x^n$

$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$ $\quad y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - x^2 \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - 2x \sum_{n=1}^{\infty} n a_n x^{n-1} + 6\sum_{n=0}^{\infty} a_n x^n =$

$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=2}^{\infty} n(n-1) a_n x^n - \sum_{n=1}^{\infty} 2n a_n x^n + \sum_{n=0}^{\infty} 6 a_n x^n =$

$[2a_2 + 6a_0] x^0 + [6a_3 - 2a_1 + 6a_1] x^1 +$

$\sum_{n=2}^{\infty} [(n+2)(n+1)a_{n+2} - n(n-1)a_n - 2n a_n + 6a_n] x^n = 0$

$\Rightarrow 2a_2 + 6a_0 = 0 \quad \Rightarrow a_2 = -3a_0$

$\Rightarrow 6a_3 + 4a_1 = 0 \quad \Rightarrow a_3 = \frac{-2}{3} a_1$

$n = 0, 1, 2, \ldots$

$\Rightarrow (n+2)(n+1)a_{n+2} - (n(n-1) + 2n - 6)a_n = 0 \Rightarrow a_{n+2} = \frac{n(n-1) + 2n - 6}{(n+2)(n+1)} a_n$

$= \frac{(n+3)(n-2)}{(n+2)(n+1)} a_n$

$a_4 = \frac{(5)(0)}{(4)(3)} a_2 = 0$

$a_5 = \frac{(6)(1)}{(5)(4)} a_3 = \frac{6}{20} \cdot \frac{-2}{3} a_1 = \boxed{\frac{-1}{5} a_1}$

$a_6 = \frac{(7)(2)}{(6)(5)} a_4 = 0$

$\Longrightarrow$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6$$

$$= a_0 + a_1 x + (-3a_0)x^2 + \left(\frac{-2}{3}a_1\right)x^3 + 0 + \left(\frac{-1}{5}a_1\right)x^5 + 0$$

$$= a_0(1-3x^2) + a_1\left(x - \frac{2}{3}x^3 - \frac{1}{5}x^5 - \cdots\right)$$

Polynomial solution

$$y_1 = 1-3x^2$$

to find the second poly. solution use the wronskian

$$y'' - \frac{2x}{1-x^2}y' + \frac{6}{1-x^2}y = 0$$

$$W(y_1, y_2) = c\,e^{-\int \frac{2x}{1-x^2}dx} = c\,e^{\int \frac{2x}{x^2-1}dx} = e^{\ln|x^2-1|} = c(x^2-1)$$

$$\begin{vmatrix} 1-3x^2 & y_2 \\ -6x & y_2' \end{vmatrix} = x^2-1$$

$$(1-3x^2)y_2' + (6x)y_2 = x^2-1$$

$$y_2' + \frac{6x}{1-3x^2}y_2 = \frac{x^2-1}{1-3x^2}$$

$$\mu(t) = e^{\int p(t)} = e^{\int \frac{6x}{1-3x^2}} = e^{-\ln|1-3x^2|} = \frac{1}{1-3x^2}$$

$\vdots$

(7)

a) $(x+2)^2 y'' - 6y = 0$ $\quad y(-1)=1 \quad y'(-1)=1$

find $y(0)$

Euler equation

$r^2 - r - 6 = 0 \Rightarrow r=3 \quad r=-2$

$y(x) = c_1 |x+2|^3 + c_2 |x+2|^{-2}$ $\qquad y' = 3c_1|x+2|^2 + -2c_2|x+$

$y(-1)=1 \Rightarrow (c_1 + c_2 = 1)2$

$y'(-1)=1 \Rightarrow 3c_1 - 2c_2 = 1$

$5c_1 = 3 \Rightarrow c_1 = \frac{3}{5}$

$\frac{3}{5} + c_2 = 1 \Rightarrow c_2 = \frac{2}{5}$

$y(t) = \frac{3}{5}|x+2|^3 + \frac{2}{5}|x+2|^{-2}$

$y(0) = \frac{3}{5}(8) + \frac{2}{5}\cdot\frac{1}{4}$

$= \frac{24}{5} + \frac{1}{10} = \frac{49}{5}$

(b) $L(f(t)) = L(u_1(t)\sin 2t)$

$= L(u_1(t)\sin[2(t-1)+2])$

$= L(u_1(t)[\sin 2(t-1)\cos 2 + \sin 2\cos 2(t-1)])$

$= \left(\cos 2\, e^{-s}\frac{2}{s^2+4} + \sin 2\, e^{-s}\frac{s}{s^2+4}\right)$

$L(f(t)) = \dfrac{(1+3s)e^{-s}}{(s-2)^2+9}$ $\quad f(2)$?

$L(f(t)) = \dfrac{e^{-s}}{(s-2)^2+9} + \dfrac{3e^{-s}s}{(s-2)^2+9}$

$f(t) = u_1(t)e^{2(t-1)}\dfrac{\sin 3(t-1)}{3} + 3u_1(t)e^{2(t-1)}\cos 3(t-1)$

(Q 4)

Proof

$L[u_a(t)f(t-a)] = e^{-as}L(f(t))$

$\Rightarrow \int_0^\infty e^{-st}u_a(t)f(t-a)\,dt$

$= \int_a^\infty e^{-st}f(t-a)\,dt$ $\quad$ let $x = t-a$, $dx = dt$

$= \int_0^\infty e^{-s(x+a)}f(x)\,dx$ $\quad$ $t=a \Rightarrow x=0$, $t=\infty \Rightarrow x=\infty$

$= \int_0^\infty e^{-sx}\cdot e^{-sa}f(x)\,dx$

$= e^{-sa}\int_0^\infty e^{-sx}f(x)\,dx$

$= e^{-as}L(f(x))$

$= e^{-as}L(f(t))$

$2e^{t} - t^2e^{0} + 2te^{0} - 2e^{0}$ $(e^{t} * t^2)$

$2e^t - t^2 + 2t - 2$

$= \int_0^t f(t-u)g(u)du$

$2e - 1 - 2 - 2$

$\boxed{2e-5}$

$= \int_0^t (t-u)^2 e^u du$

$(t-u)^2 e^u + 2(t-u)e^u + 2e^u \Big|_0^t$

$(t-u)^2$ $+$ e^u

$-2(t-u)$ $-$ e^u

2 $+$ e^u

Dohu shachu

$\dfrac{2e^t - 2e^0}{2e^t - 2}$